4.4 Classical DSB-SC Modulators

To produce the modulated signal $A_c \cos(2\pi f_c t) m(t)$, we may use the following methods which generate the modulated signal along with other signals which can be eliminated by a bandpass filter restricting frequency contents to around f_c.

4.50. Multiplier Modulators [5, p 184] or **Product Modulator**[3, p 180]: Here modulation is achieved directly by multiplying $m(t)$ by $\cos(2\pi f_c t)$ using an analog multiplier whose output is proportional to the product of two input signals.

- Such a multiplier may be obtained from
  (a) a variable-gain amplifier in which the gain parameter (such as the the β of a transistor) is controlled by one of the signals, say, $m(t)$. When the signal $\cos(2\pi f_c t)$ is applied at the input of this amplifier, the output is then proportional to $m(t)\cos(2\pi f_c t)$.
  (b) two logarithmic and an antilogarithmic amplifiers with outputs proportional to the log and antilog of their inputs, respectively.
    - Key equation:
$$A \times B = e^{(\ln A + \ln B)}.$$

4.51. Square Modulator: When it is easier to build a squarer than a multiplier, use

$$\begin{aligned}(m(t) + c\cos(\omega_c t))^2 &= m^2(t) + 2c\,m(t)\cos(\omega_c t) + c^2\cos^2(\omega_c t)\\ &= m^2(t) + 2c\,m(t)\cos(\omega_c t) + \frac{c^2}{2} + \frac{c^2}{2}\cos(2\omega_c t).\end{aligned}$$

Using a band-pass filter (BPF) whose frequency response is

$$H_{BP}(f) = \begin{cases} g, & |f - f_c| \le B, \\ g, & |f - (-f_c)| \le B, \\ 0, & \text{otherwise}, \end{cases}$$

we can produce $2cgm(t)\cos(\omega_c t)$ at the output of the BPF. Choosing the gain g to be $(c\sqrt{2})^{-1}$, we get $m(t) \times \sqrt{2}\cos(\omega_c t)$.

- Alternative, can use $\left(m(t) + c\cos\left(\frac{\omega_c}{2}t\right)\right)^3$.

4.52. Another conceptually nice way to produce a signal of the form $A_c m(t)\cos(\omega_c t)$ is to (1) multiply $m(t)$ by "any" periodic and even signal $r(t)$ whose period is $T_c = \frac{2\pi}{\omega_c}$ and (2) pass the result though a BPF. Because $r(t)$ is an even function, we know that

$2\pi(kf_c)t$

$$r(t) = c_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_c t) \text{ for some } c_0, a_1, a_2, \ldots.$$

Therefore,

$$m(t)r(t) = c_0 m(t) + \sum_{k=1}^{\infty} a_k m(t)\cos(k\omega_c t).$$

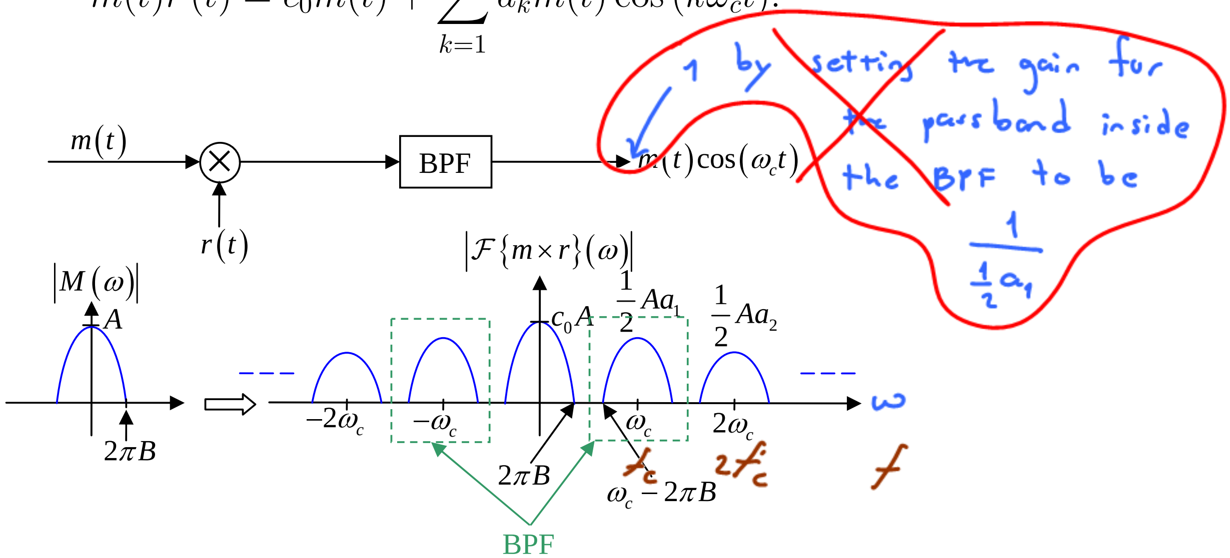


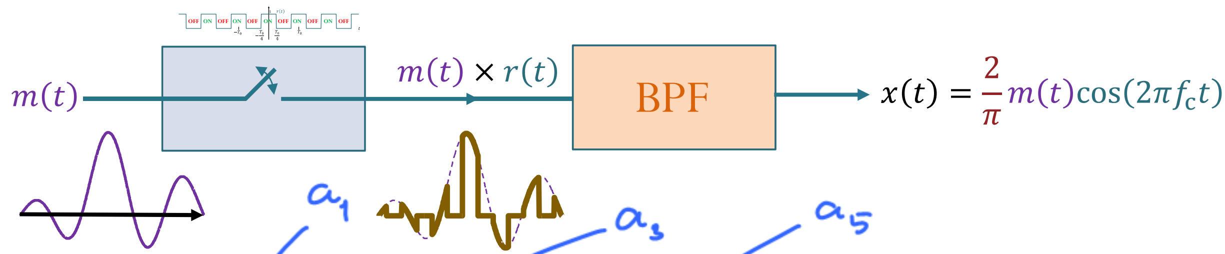
Figure 20: Modulation of $m(t)$ via even and periodic $r(t)$

See also [4, p 157]. In general, for this scheme to work, we need

- $a_1 \neq 0$; that is T_c is the "least" period of r;
- $f_c > 2B$ (to prevent overlapping).

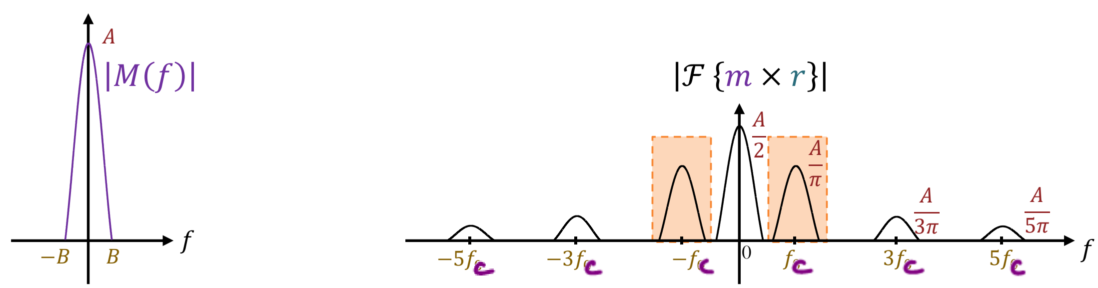
Note that if $r(t)$ is not even, then by (43c), the outputted modulated signal is of the form $a_1 m(t)\cos(\omega_c t + \phi_1)$.

4.53. ***Switching modulator***: An example of a periodic and even function $r(t)$ is the square pulse train considered in Example 4.43. Recall that multiplying this $r(t)$ to a signal $m(t)$ is equivalent to switching $m(t)$ on and off periodically.

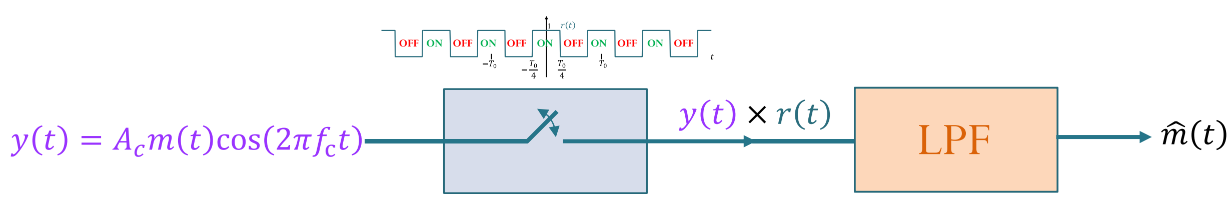


$$r(t) = \frac{1}{2} + \overbrace{\frac{2}{\pi}}^{a_1}\cos(2\pi f_c t) - \overbrace{\frac{2}{3\pi}}^{a_3}\cos(2\pi(3f_c)t) + \overbrace{\frac{2}{5\pi}}^{a_5}\cos(2\pi(5f_c)t) + \cdots$$

$$m(t) \times r(t) = \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) - \frac{2}{3\pi}m(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}m(t)\cos(2\pi(5f_c)t) + \cdots$$



4.54. ***Switching Demodulator***: The switching technique can also be used at the demodulator as well.



We have seen that, for DSB-SC modem, the key equation is given by (34). When switching demodulator is used, the key equation is

$$\text{LPF}\{\underbrace{A_c m(t)\cos(\omega_c t)}_{\substack{y(t) \\ \text{(the received} \\ \text{signal)}}} \times \underbrace{1[\cos(\omega_c t) \geq 0]}_{\substack{\text{the ON-OFF} \\ \text{operation}}}\} = \frac{A_c}{\pi}m(t) \qquad (52)$$

[4, p 162].

$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(2\pi(3f_c)t) + \frac{2}{5\pi}\cos(2\pi(5f_c)t) + \cdots$$

$$y(t)r(t) = \frac{1}{2}y(t) + \frac{2}{\pi}y(t)\cos(2\pi f_c t) - \frac{2}{3\pi}y(t)\cos(2\pi(3f_c)t) + \frac{2}{5\pi}y(t)\cos(2\pi(5f_c)t) + \cdots$$

$$\begin{aligned}
&= \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \\
&\quad + \frac{2}{\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t) \\
&\quad - \frac{2}{3\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi(3f_c)t) \\
&\quad + \frac{2}{5\pi}A_c m(t)\cos(2\pi f_c t)\cos(2\pi(5f_c)t) + \cdots
\end{aligned}$$

$$\cos(A)\cos(B) = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$$

$$\begin{aligned}
&= \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \\
&\quad + \frac{1}{\pi}A_c m(t)\left(1 + \cos(2\pi(2f_c)t)\right) \\
&\quad - \frac{1}{3\pi}A_c m(t)\left(\cos(2\pi(2f_c)t) + \cos(2\pi(4f_c)t)\right) \\
&\quad + \frac{1}{5\pi}A_c m(t)\left(\cos(2\pi(4f_c)t) + \cos(2\pi(6f_c)t)\right) + \cdots
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2}A_c m(t)\cos(2\pi f_c t) \\
&\quad \boxed{+\frac{1}{\pi}A_c m(t)} + \frac{1}{\pi}A_c m(t)\cos(2\pi(2f_c)t) \\
&\quad - \frac{1}{3\pi}A_c m(t)\cos(2\pi(2f_c)t) - \frac{1}{3\pi}A_c m(t)\cos(2\pi(4f_c)t) \\
&\quad + \frac{1}{5\pi}A_c m(t)\cos(2\pi(4f_c)t) + \frac{1}{5\pi}A_c m(t)\cos(2\pi(6f_c)t) + \cdots
\end{aligned}$$

Note that this technique still requires the switching to be in sync with the incoming cosine as in the basic DSB-SC.